New paradox in the special theory of relativity generated by the string dynamics

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Abstract

It is proved that the definition of sinultaneity by Einstein leads to the paradox motion of the string from the viewpoint of the observer in the inertial system S' moving with velocity v with regard to the inertial system S.

Key words. Special theory of relativity, string, simultaneity, wave equation.

Einstein proved in his well known article and book (Einstein, 1905; 1919; 1922) that the simultaneity in the inertial system S' moving with velocity v with regard to the inertial system S is broken. Einstein writes (Einstein, 1905):

So we see that we cannot attach any absolute signification to the concept of simultaneity, but that two events which, viewed from a system of co-ordinates, are simultaneous, can no longer be looked upon as simultaneous events when envisaged from a system which is in motion relatively to that system.

Let us show that the Einstein realization of simultaneity leads to the paradox if we consider the dynamics of the elastic string.

So, let the string in the system S with the equilibrium length l_o is elongated to the length l, $l > l_o$, and it is fixed et the ends. At time $t = 0$ the ends of he string are released. The string motion is described by the wave equation

$$\frac{1}{a^2}\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\partial^2 u(x,t)}{\partial x^2} = 0 \tag{1}$$

including the initial conditions

$$u(x,0) = Ax; \quad \dot{u}(x,0) = 0 \tag{2}$$

and the boundary conditions expressing the fact that the end of the string are free from time $t > 0$. Here $a \neq c$, c being the velocity of light.

Without mathematics we know that the center of mass of string is at the rest. This result can be immediately confirmed using the rubber string with the fixation of ends by fingers. In other words, by the string-finger experiment. The experiment with the rubber string is very simple and can be performed everywhere.

The situation in the system S' moving with the velocity v with regard to the system S is different because the releasing of the ends of the rubber string is not simultaneous according the Einstein definition of simultaneity. It means that the motion of the string is a such that the center of mass changes its velocity. We also can verify the change of velocity of the center of mass by the rubber string experiment with the noninstantaneous releasing of the ends of the string.

So, we have a paradox. The center of mass of he string in system S does not change its state of motion, while in the system S' it does. This paradox is not involved in the collection of paradoxes of relativity (Goldblatt, 1972; Terletzkii, 1966) and in the relativistic paradoxes in American Journal of Physics. To our knowledge, this paradox is not involved in any monograph of the string theory.

There is no doubt that solution of this paradox will refresh the interest in the theory of real strings described by the equations of mathematical physics.

Let us remark only that the paradox can be resolved by observing that the wave equation (1) is not relativistically invariant with regard to the Lorentz transformation

$$x' = \gamma(x - vt), \quad t' = \gamma(t - (v/c^2)x); \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{3}$$

It means that it is necessary to transform equation (1) to the system S' and then to solve the motion of strings with the broken boundary condition. To our knowledge the solution of this problem was not published till this time.

The resolution of the paradox with the broken simultaneity is important not only from the viewpoint of relativity but from the viewpoint of pedagogical thinking and from the viewpoint of the methodology of science. The discussion of such problems as simultaneity and string motion in the different inertial systems is important for the logical foundation of science and it means that every elementary new information, if true, is of great influence on science.

Let us remark that we get another paradox if we consider equation (1) with the boundary conditions

$$u(0, t) = 0; \quad u(l, t) = 0 \tag{4}$$

and the initial conditions in the general form

$$u(x, 0) = f(x); \quad \dot{u}(x, 0) = g(x) \tag{5}$$

where f, g are arbitrary functions.

It is well known that the general solution is of the form

$$u = \varphi(t + x/a) + \psi(t - x/a) \tag{6}$$

However, the solution of such vibration of the string is also of the form (which was known also to Daniel Bernoulli and Leonhard Euler):

$$u = \sum_{k=1}^{\infty} \left(a_k \cos \frac{k\pi a t}{l_0} + b_k \sin \frac{k\pi a t}{l_0} \right) \sin \frac{k\pi x}{l_0} \qquad (7)$$

If we transform the solution (6) to the system S', then we transform only coordinate x and time t. On the other hand, if we transform the solution (7), then we must also respect the Lorentz contraction of l_0. And we are not sure, a priori, that the two results will be identical. And this is the paradox.

References.

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